

Probability outcomes: lists, trees and fair comparisons

Compare your reasoning, not just the final result. Equivalent correct methods are welcome.

Public practice with published answers. This is not a full-topic readiness assessment.

1. A fair standard die has faces 1 to 6. List the outcomes greater than 4 and find their probability.

Worked solution 1: Outcomes 5 and 6; probability $\frac{1}{3}$.

1. There are six equally likely faces.
2. Two faces are greater than 4: 5 and 6.
3. Probability = $\frac{2}{6} = \frac{1}{3}$.

2. A bag contains 3 orange, 2 purple and 1 white counter, identical except for colour. Mix well and choose one without looking. Find the probability of purple. Are the three colours equally likely? Explain.

Worked solution 2: $P(\text{purple}) = \frac{1}{3}$. The colours are not equally likely.

1. There are $3 + 2 + 1 = 6$ counters, each assumed equally likely to be selected.
2. Purple has 2 favourable counters, so $\frac{2}{6} = \frac{1}{3}$.
3. Orange has probability $\frac{3}{6}$ and white $\frac{1}{6}$. Counting three colour names as three equal outcomes would be incorrect.

3. Toss a fair coin and independently spin a fair three-sector spinner labelled 1, 2, 3. List all ordered outcomes and find the probability of heads with an odd spinner number.

Worked solution 3: H1, H2, H3, T1, T2, T3; probability $\frac{1}{3}$.

1. Pair each coin result with all three spinner results to list six equally likely outcomes.
2. H1 and H3 are favourable: heads and an odd number.
3. Probability = $\frac{2}{6} = \frac{1}{3}$, also $(\frac{1}{2}) \times (\frac{2}{3})$.

4. A bag has 4 green and 2 yellow identical counters. Draw one at random, replace it, mix, then draw again. Draw a probability tree and find the probability of two green counters.

Worked solution 4: $\frac{4}{9}$.

1. At each draw, green has probability $\frac{4}{6} = \frac{2}{3}$ and yellow $\frac{2}{6} = \frac{1}{3}$.
2. Replacement restores the original counts, so each first branch has the same second-stage probabilities.
3. The green–green path has probability $(\frac{2}{3}) \times (\frac{2}{3}) = \frac{4}{9}$.

5. Repeat the experiment with 4 green and 2 yellow counters, now WITHOUT replacement. Draw the green-first branches, find the probability of two greens, and compare it with $\frac{4}{9}$ for replacement.

Worked solution 5: $\frac{2}{5}$, which is $\frac{2}{45}$ less than $\frac{4}{9}$.

1. First green: $\frac{4}{6}$. After a green is removed, 3 green and 2 yellow remain.
2. On the green-first branch, second green is $\frac{3}{5}$ and second yellow is $\frac{2}{5}$.
3. $P(\text{two greens}) = (\frac{4}{6}) \times (\frac{3}{5}) = \frac{12}{30} = \frac{2}{5}$.
4. $\frac{4}{9} - \frac{2}{5} = \frac{20}{45} - \frac{18}{45} = \frac{2}{45}$. Removing a green lowers the next green probability.

6. Draw two counters without replacement from the same bag of 4 green and 2 yellow counters. Find the probability of one of each colour in either order. A class records one of each in 31 of 60 repeated trials, resetting the bag each trial. Compare the experimental proportion with the theoretical probability. Does a difference prove the model is wrong?

Worked solution 6: Theoretical probability $\frac{8}{15}$; experimental proportion $\frac{31}{60}$, which is $\frac{1}{60}$ lower. The difference alone does not prove the model is wrong.

1. Green then yellow: $(\frac{4}{6}) \times (\frac{2}{5}) = \frac{4}{15}$.
2. Yellow then green: $(\frac{2}{6}) \times (\frac{4}{5}) = \frac{4}{15}$.
3. The two orders are separate outcomes, so add: $\frac{4}{15} + \frac{4}{15} = \frac{8}{15} = \frac{32}{60}$.
4. Experimental proportion is $\frac{31}{60}$, one trial out of 60 below $\frac{32}{60}$.
5. Random variation can produce a difference. Check the procedure and collect more trials; even a larger sample need not match theory exactly.